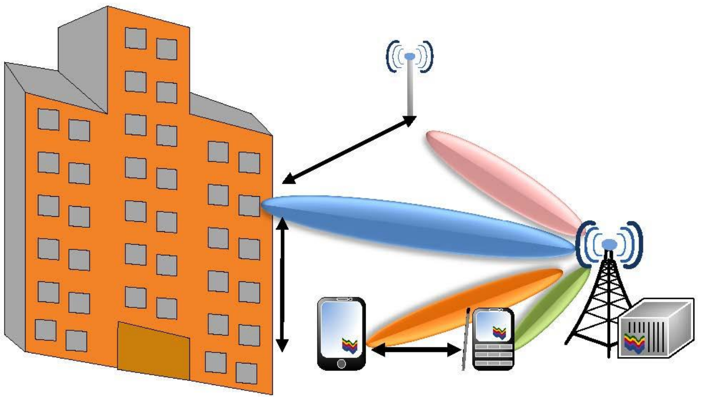
# Introduction to Massive MIMO

**Hongrui Wang**
**hongruiseu@gmail.com**

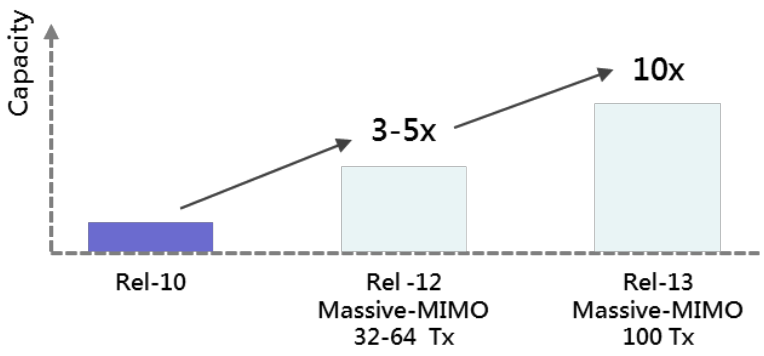
# Introduction

- **What is Massive MIMO?**



- Have hundreds of antennas
  - Increase throughput
- Enables 3D beamforming
  - Suppress Inter-cell interference
- Multiuser MIMO support

- **Benefit of Massive Antenna**
  - Capacity enhancement[1]




- Mathematically Analysis:
  - Required **Tx energy/bit** is arbitrarily **small**
  - **Eliminate** the effects of uncorrelated noise
  - **Compensate** poor-quality CSI

[1] RWS-120002, "Technologies for Rel-12 and Onwards," Samsung, 3GPP RAN WS on Rel-12 and onwards, Ljubljana, Slovenia, June 11–12, 2012.

# Introduction

- **System Model of Massive MIMO[2]**
  - Consider a $M \times K$ MIMO

$$\mathbf{y}_{M\times 1} = \mathbf{H}_{M\times K}\mathbf{x}_{K\times 1} + \mathbf{n}_{M\times 1}$$

  - BS process by matched filtering

$$\mathbf{y} \Rightarrow \frac{1}{M}\mathbf{H}^H\mathbf{y} = \frac{1}{M}\mathbf{H}^H\mathbf{H}\mathbf{x} + \frac{1}{M}\mathbf{H}^H\mathbf{n}$$

$$\frac{1}{M}\mathbf{H}^H\mathbf{H} = \begin{bmatrix} \frac{\|\mathbf{h}_1\|^2}{M} & \frac{\mathbf{h}_1^H\mathbf{h}_2}{M} & \cdots & \frac{\mathbf{h}_1^H\mathbf{h}_K}{M} \\ \frac{\mathbf{h}_2^H\mathbf{h}_1}{M} & \frac{\|\mathbf{h}_2\|^2}{M} & & \frac{\mathbf{h}_2^H\mathbf{h}_K}{M} \\ \vdots & & \ddots & \vdots \\ \frac{\mathbf{h}_K^H\mathbf{h}_1}{M} & \frac{\mathbf{h}_K^H\mathbf{h}_2}{M} & \cdots & \frac{\|\mathbf{h}_K\|^2}{M} \end{bmatrix}$$

as $M \to \infty$

$\xrightarrow[M\to\infty]{a.s.} 0$

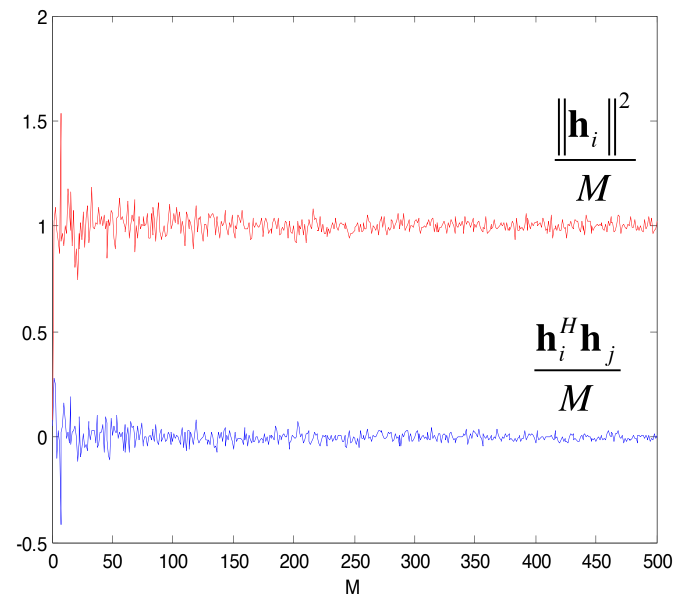
$\xrightarrow[M\to\infty]{a.s.} 1$

**By strong law of large numbers**

- By strong law, we have

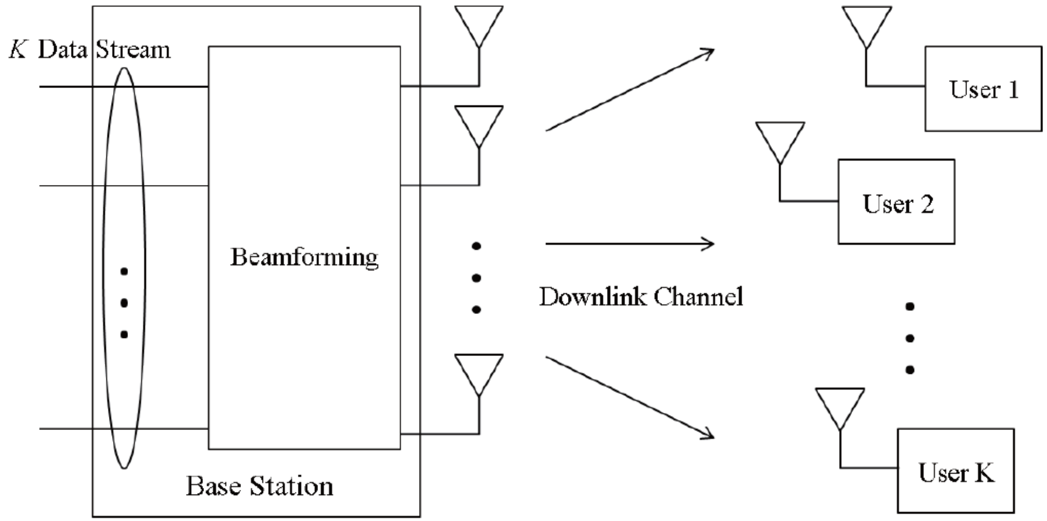
$$\frac{1}{M}\mathbf{H}^H\mathbf{y} \xrightarrow[M\to\infty,\ K=\text{const.}]{a.s.} \mathbf{x}$$

  - Interference and noise vanish
  - The matched filter is optimal
  - Transmit power can be small




[2] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," IEEE Trans. Wireless Commun., vol. 9, no. 11, pp. 3590–3600, Nov. 2010.

# Single-cell Massive MIMO (DL) (1)




- Parameters
  - $\mathbf{h}_k$ : small scale fading
  - $\mathbf{w}_k$ : beamforming vector
  - $M$ : # of BS antenna
  - $K$: # of users
  - Single antenna at user
- Received vector

$$\mathbf{y}_{K\times 1} = \sqrt{p_d}\,\mathbf{H}_{K\times M}\mathbf{x}_{M\times 1} + \mathbf{n}_{K\times 1} = \sqrt{p_d}\,\mathbf{H}\mathbf{W}_{M\times K}\mathbf{s}_{K\times 1} + \mathbf{n}$$

- Received signal after linear precoding: (MRT $\mathbf{W} = \mathbf{H}^H$ )

$$y_k = \underbrace{\sqrt{p_d}\,\mathbf{h}_k \mathbf{w}_k s_k}_{\text{desired signal}} + \underbrace{\sqrt{p_d} \sum_{i=1, i\neq k}^{K} \mathbf{h}_k \mathbf{w}_i s_i}_{\text{interference}} + \underbrace{n_k}_{\text{noise}}$$

# Single-cell Massive MIMO (DL) (2)

- Deterministic form of the $\mathrm{SINR}_k / R_{\mathrm{sum}}$ as $M$, $K \to \infty$, $M/K = \alpha$
  - MRT : $\frac{1}{K} \sum_{i=1, i \neq k}^{K} \left| \mathbf{h}_k \mathbf{h}_i^H \right|^2 \approx E\left\{ \left| \mathbf{h}_k \mathbf{h}_i^H \right|^2 \right\} = M \ \left( \because \left| \mathbf{h}_k \mathbf{h}_i^H \right|^2 \sim \chi_M^2 \right), \ \gamma = \left\| \mathbf{H}^H \right\|_F^2 \approx KM$
  - SINR of the kth user :

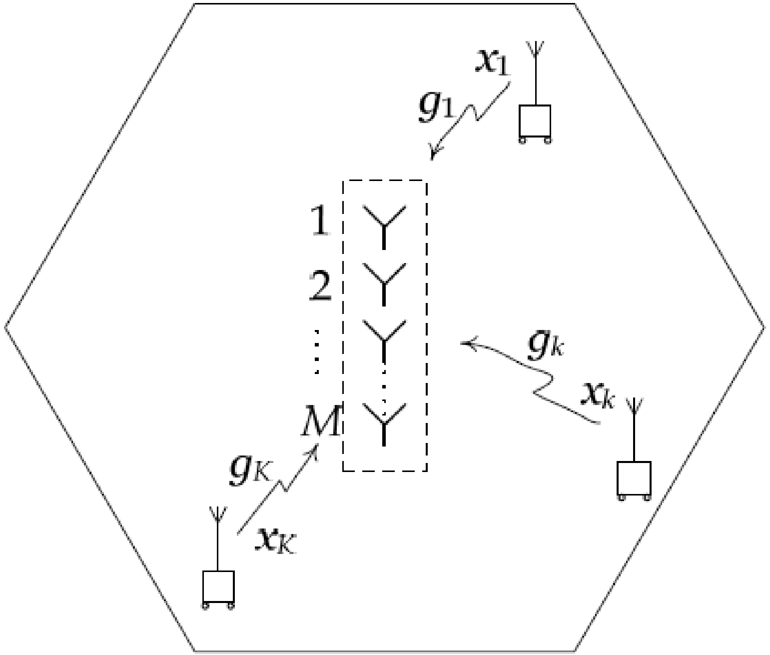
$$\mathrm{SINR}_k^{mrt} = \frac{\frac{p_d}{\gamma} \left| \mathbf{h}_k \mathbf{h}_k^H \right|^2}{\frac{p_d}{\gamma} \sum_{i=1, i \neq k}^{K} \left| \mathbf{h}_k \mathbf{h}_i^H \right|^2 + 1} \xrightarrow{a.s.} \frac{p_d \alpha}{p_d + 1} \text{ as } M, K \to \infty$$

  - Ergodic sum rate :

$$R_{\mathrm{sum}}^{mrt} = \sum_{k=1}^{K} E\{R_k\} = K \cdot \log_2 \left( 1 + \frac{p_d \alpha}{p_d + 1} \right)$$

# Single-cell Massive MIMO (UL) (1)

- Parameters
  - $\mathbf{g}_k = \sqrt{\beta_k}\mathbf{h}_k$
  - $\mathbf{h}_k$ : small scale fading
  - $\beta_k$ : path loss + shadowing
  - SNR for the *k*th UE: $p_u \beta_k$
  - $\mathbf{G} = \mathbf{H}\mathbf{D}^{1/2}, \quad \mathbf{D} \triangleq \mathrm{diag}\left(\beta_1, \ldots, \beta_K\right)$




- Received signal

$$\mathbf{y}_{M\times 1} = \sqrt{p_u}\mathbf{G}_{M\times K}\mathbf{x}_{K\times 1} + \mathbf{n}_{M\times 1}$$

- Received signal after the linear detector

$$\mathbf{r} = \sqrt{p_u}\mathbf{A}^H\mathbf{G}\mathbf{x} + \mathbf{A}^H\mathbf{n} \rightarrow r_k = \sqrt{p_u}\mathbf{a}_k^H\mathbf{G}\mathbf{x} + \mathbf{a}_k^H\mathbf{n} = \underbrace{\sqrt{p_u}\mathbf{a}_k^H\mathbf{g}_k x_k}_{\text{desired signal}} + \underbrace{\sqrt{p_u}\sum_{i=1, i\neq k}^{K}\mathbf{a}_k^H\mathbf{g}_i x_i}_{\text{interference}} + \underbrace{\mathbf{a}_k^H\mathbf{n}}_{\text{noise}}$$

# Single-cell Massive MIMO (UL) (2)

- Uplink performance with MRC – Perfect CSI

## Capacity lower bound

$$R_{P,k}^{\mathrm{mrc}} = E\left\{\log_2\left(1+\frac{p_u\|g_k\|^4}{p_u\sum_{i=1,i\neq k}^{K}\left|g_k^H g_i\right|^2+\|g_k\|^2}\right)\right\}$$

$$\geq \log_2\left(1+\left(E\left\{\frac{p_u\sum_{i=1,i\neq k}^{K}\left|g_k^H g_i\right|^2+\|g_k\|^2}{p_u\|g_k\|^4}\right\}\right)^{-1}\right)$$

$$= \log_2\left(1+\frac{p_u(M-1)\beta_k}{p_u\sum_{i=1,i\neq k}^{K}\beta_i+1}\right) \triangleq \tilde{R}_{P,k}^{\mathrm{mrc}}$$

$p_u = E_u/M$

## Limit case

$$\tilde{R}_{P,k}^{\mathrm{mrc}} = \log_2\left(1+\frac{\frac{E_u}{M}(M-1)\beta_k}{\frac{E_u}{M}\sum_{i=1,i\neq k}^{K}\beta_i+1}\right)$$

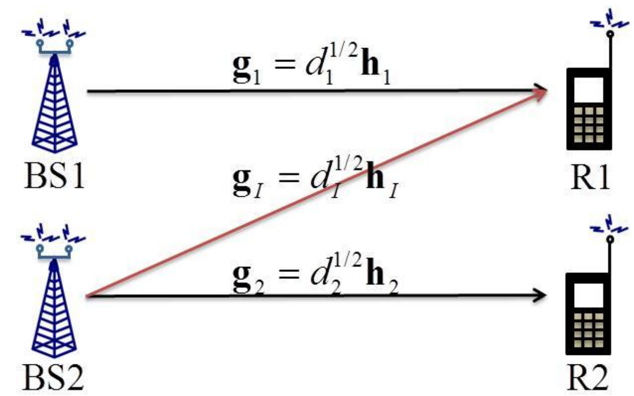
$$\rightarrow \log_2\left(1+\beta_k E_u\right) \text{ as } M\rightarrow\infty$$

- **Small-scale fading/ Inter-user interference goes away in the limit**
- **Tx power can be scaled as** $\propto 1/M$

# Multi-Cell Massive MIMO : Inter-Cell Interference

- Non-cooperative multi-cell environment
  - Received signal for R1

$$y_1 \approx \sqrt{\frac{p_d}{M}} d_1^{1/2} \mathbf{h}_1 \mathbf{h}_1^H \tilde{x}_1 + \sqrt{\frac{p_d}{M}} d_I^{1/2} \mathbf{h}_I \mathbf{h}_2^H \tilde{x}_2 + \mathbf{n}$$




  - Scaling by $1/\sqrt{M}$ at received signal $y_1$

$$\frac{1}{\sqrt{M}} y_1 \approx \sqrt{p_d} d_1^{1/2} \frac{\mathbf{h}_1 \mathbf{h}_1^H}{M} \tilde{x}_1 + \sqrt{p_d} d_I^{1/2} \frac{\mathbf{h}_I \mathbf{h}_2^H}{M} \tilde{x}_2 + \frac{1}{\sqrt{M}} \mathbf{n} \rightarrow 0$$

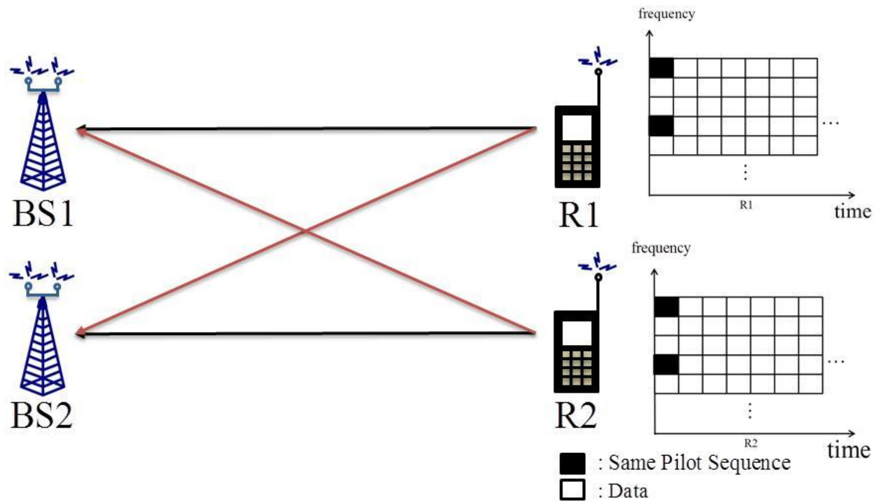
  - For large MIMO (large M),

$$\frac{\|\mathbf{h}_i\|^2}{M} = \frac{|h_1^i|^2 + \cdots + |h_M^i|^2}{M} \approx \underbrace{\mathrm{Var}[h]}_{1} + \underbrace{(E[h])^2}_{0} = 1$$

$$\frac{\mathbf{h}_i \mathbf{h}_j^H}{M}_{(i \neq j)} = \frac{1}{M} \left( \underbrace{h_1^i h_1^{j*}}_{\text{Gaussian}} + \underbrace{h_2^i h_2^{j*}}_{\text{Gaussian}} + \cdots + \underbrace{h_M^i h_M^{j*}}_{\text{Gaussian}} \right) \approx E[h] = 0$$

# Multi-Cell Massive MIMO : Pilot Contamination

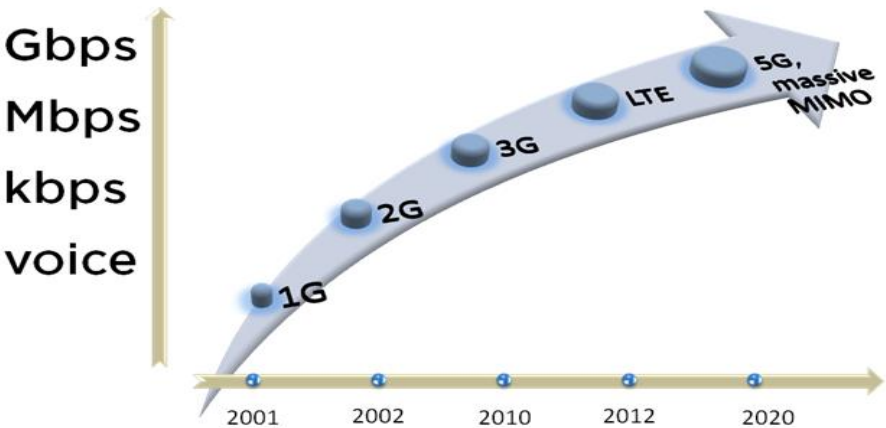
- Practical problems for non-cooperative multi-cell
  - Assumption
    - Non-cooperative multi-cell with TDD mode[3]
  - To perfectly mitigate interference at large *M*, exact MRT scheme is needed
  - Perfect CSIT is necessary to design exact MRT precoder
    - UL pilots are allocated in same time-frequency elements to obtain perfect CSIT
    - UL pilots can be separated by orthogonal sequences → Can obtain perfect CSIT
  - Problem: Exact same time-frequency elements with same pilot sequence
    - Perfect CSIT is impossible → **pilot contamination problem**
  - Interference (inter-cell & intra-cell) cannot be vanished even $M \to \infty$




[3] F. Rusek et al., "Scailing up MIMO: Opportunities and challenges with very large arrays,"

# Summery of Massive MIMO : a key technology to 5G

- METIS, the EU funded organisation, defines '5G' supporting:
  - 1000 times higher mobile data volume per area
  - 10 times to 100 times higher typical user data rate




**Further capacity enhancement is needed**

**Massive MIMO is POWERFUL!**

- In 5G, massive MIMO will work with other technologies together[4] :
  - Coexistence with Small Cells
  - Coexistence with mmWare

[4] Jeffrey G. Andrews, " What will 5G be?", IEEE JSAC SPECIAL ISSUE ON 5G WIRELESS COMMUNICATION SYSTEMS